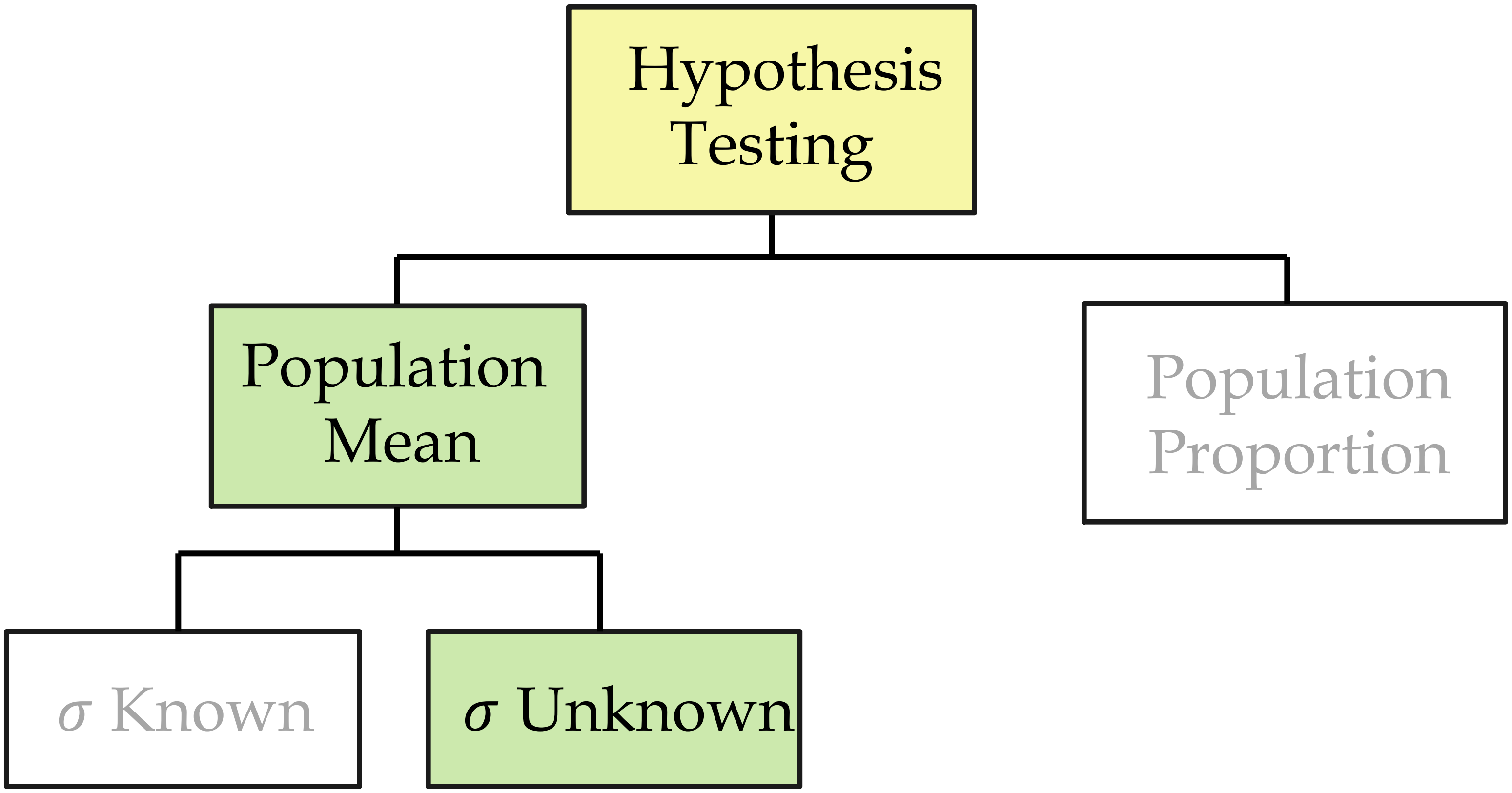
Hypothesis Testing for a Single Population

- Hypothesis Testing for the Population Mean
  - σ is Known (already discussed)
  - σ is Unknown
- Hypothesis Testing for the Population Proportion

- Reading: Chapter 9 (except Section 9.5)

Hypothesis Testing for the Population Mean when σ is Unknown



Hypothesis Testing for the Population Mean when σ is Unknown

When the population standard deviation σ is unknown, we **substitute the sample standard deviation**, s, in place of σ

- The sample standard deviation can be computed once a sample has been collected

We use **the Student's *t*-distribution** with $df = n - 1$ to find the critical value and the p-value rather than the standard normal distribution

Hypothesis testing procedure is valid only if the sampling distribution of $\bar{x}$ is normal:

- The population data follow the normal distribution
- Sample size is large ($n \geq 30$)

One-Tail Hypothesis Test for the Population Mean (σ is Unknown)

Example: The average cost of a hotel room in Chicago is claimed to be \$188 per night. A travel agent thinks it is lower now.

- A random sample of 25 hotels resulted in

$$\bar{x} = \$177.50 \quad \text{and} \quad s = \$25.40$$

- Test the appropriate hypothesis using $\alpha = 0.05$ level of significance (assume the population distribution is normal)

The following slides show **steps** to complete this test

One-Tail Hypothesis Test for the Population Mean (σ is Unknown)

Step 1: Identify the null and alternative hypotheses

H_0: $\mu \geq \$188$ (status quo: average cost is not less than \$188)

H_1: $\mu < \$188$ (the cost now is less than \$188)

Step 2: Set a value for the significance level, α

- $\alpha = 0.05$ is specified for this problem

One-Tail Hypothesis Test for the Population Mean (σ is Unknown)

Step 3: Calculate the appropriate test statistic

t-test statistic for a hypothesis test for the population mean (when σ is unknown):

$$t_{\bar{x}} = \frac{\bar{x} - \mu_{H_0}}{\frac{s}{\sqrt{n}}}$$

where:

$t_{\bar{x}}$ = the t-test statistic

$\bar{x}$ = the sample mean

μ_{H_0} = hypothesized value which is assumed to be true and represents the mean of the sampling distribution

s = the sample standard deviation

n = the sample size

One-Tail Hypothesis Test for the Population Mean (σ is Unknown): Critical Value Approach

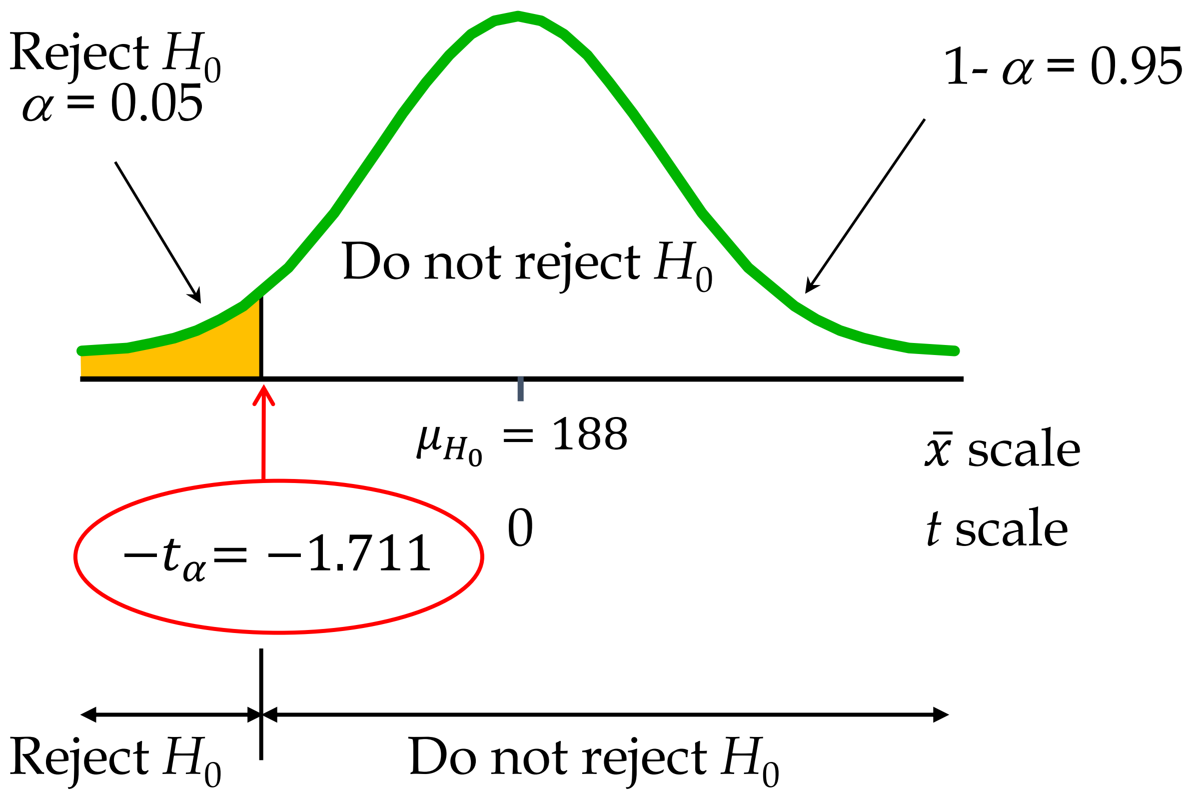
Step 3: Calculate the appropriate test statistic

$$t_{\bar{x}} = \frac{\bar{x} - \mu_{H_0}}{\frac{s}{\sqrt{n}}} = \frac{177.50 - 188}{\frac{25.40}{\sqrt{25}}} = \frac{-10.5}{5.08} = -2.07$$

Step 4: Determine the appropriate **critical value**

- σ is **unknown** $\Rightarrow$ use a t-score;
- n = 25 $\Rightarrow$ df = n - 1 = 24 degrees of freedom
- Since this is a lower-tail test, the entire area for α = 0.05 is placed on the left side of the t distribution

One-Tail Hypothesis Test for the Population Mean (σ is Unknown): Critical Value Approach



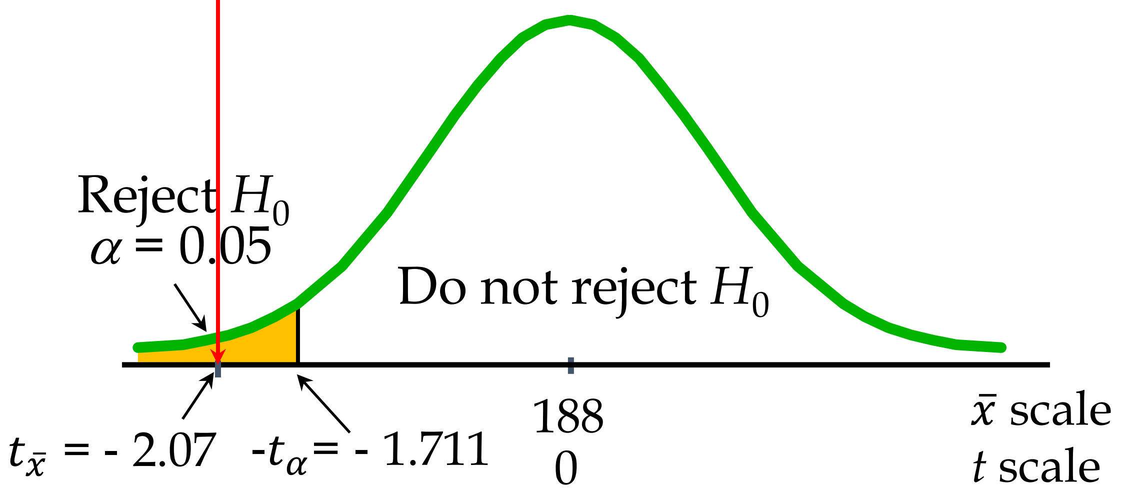
One-Tail Hypothesis Test for the Population Mean (σ is Unknown): Critical Value Approach

Step 5: Compare the test statistic $t_{\bar{x}}$ and the critical t-value

- For a one-tail (lower tail) test, reject the null hypothesis if

$$t_{\bar{x}} < -t_{\alpha}$$

- Here, $t_{\bar{x}}$ = -2.07 is less than $-t_{\alpha}$ = -1.711, so we **reject** $\boldsymbol{H_0}$



One-Tail Hypothesis Test for the Population Mean (σ is Unknown)

Conclusion

According to our sample evidence from 25 hotels, there is enough evidence to support that the average cost per night is now less than $188

One-Tail Hypothesis Test for the Population Mean: *p*-value Approach

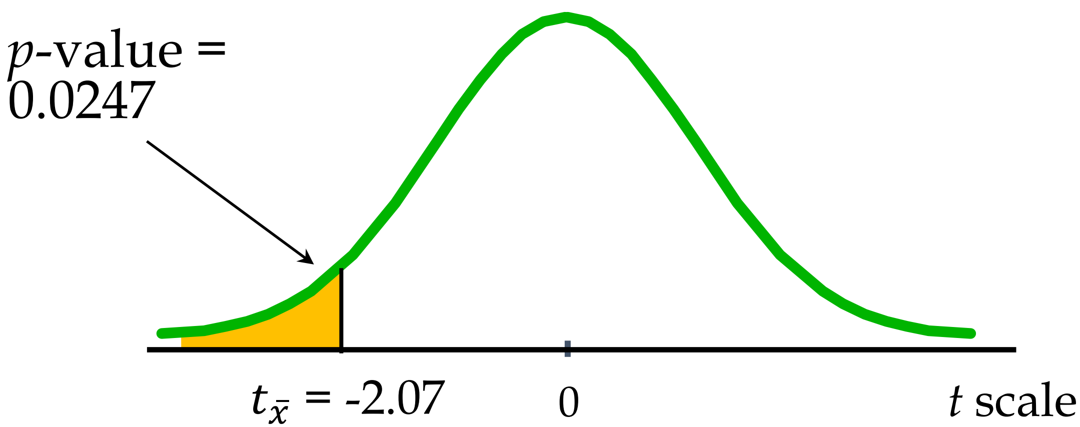
Let's use the *p*-value approach?

Step 4: Use the test statistic to compute the *p*-value

The *p*-value is a computed as the probability of obtaining *t*-value below our test statistic $t_{\bar{x}}$ = -2.07:

$$P(t < -2.07) = 0.0247$$

Note: Remember to use the appropriate *df*



One-Tail Hypothesis Test for the Population Mean: *p*-value Approach

Step 5: Compare the p-value and α

Decision Rule for Hypothesis Tests Using the p-value

$$p\text{-value} \geq \alpha \quad \Rightarrow \quad \text{Do not reject } H_0$$

$$p\text{-value} < \alpha \quad \Rightarrow \quad \text{Reject } H_0$$

Since the p-value = 0.0247 < α = 0.05, we **reject H_0**

Excel Exercise >>

Two-Tail Hypothesis Test for the Population Mean (σ Is Unknown)

Example: Claim -- the average American watches 34.5 hours of television per week

- Suppose viewing data is recorded for 10 people and the sample mean is found to be 39.6 hours and the sample standard deviation is 16.4 hours per week
- Is there enough evidence to conclude that the average hours in front of TV are different from the claim?
- Notice, that in order to proceed we need to assume that the viewing data comes from the normal distribution

The following slides show **steps** to complete this test

Two-Tail Hypothesis Test for the Population Mean (σ Is Unknown)

Step 1: Identify the null and alternative hypotheses

H_0: $\mu = 34.5$ (status quo: average time is 34.5 hours per week)

H_1: $\mu \neq 34.5$ (average viewing time is not equal to 34.5 hours per week)

Step 2: Set a value for the significance level, α

- Suppose that $\alpha = 0.02$ is chosen

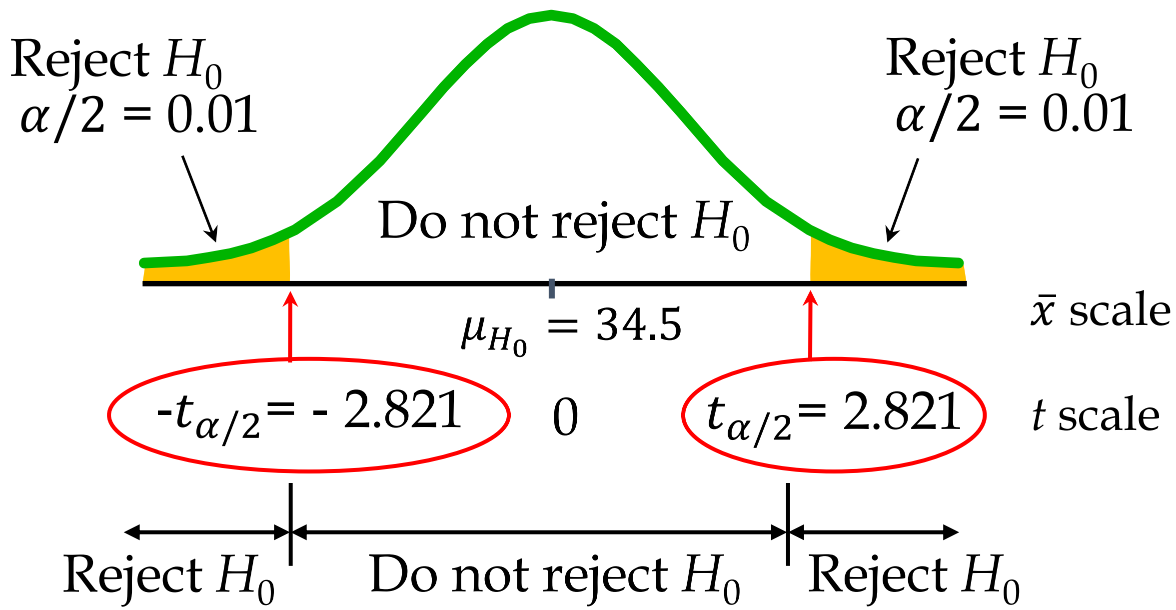
Step 3: Calculate the appropriate test statistic:

$$t_{\bar{x}} = \frac{\bar{x} - \mu_{H_0}}{\frac{s}{\sqrt{n}}} = \frac{39.6 - 34.5}{\frac{16.4}{\sqrt{10}}} = \frac{5.1}{5.19} = 0.98$$

Two-Tail Hypothesis Test for the Population Mean (σ Is Unknown)

Step 4: Determine the appropriate **critical values**

- σ is unknown $\Rightarrow$ use a t-score for $\alpha = 0.02$ and $df = n-1 = 9$
- Since this is a two-tail test, $\alpha = 0.02$ is split evenly into two tails:



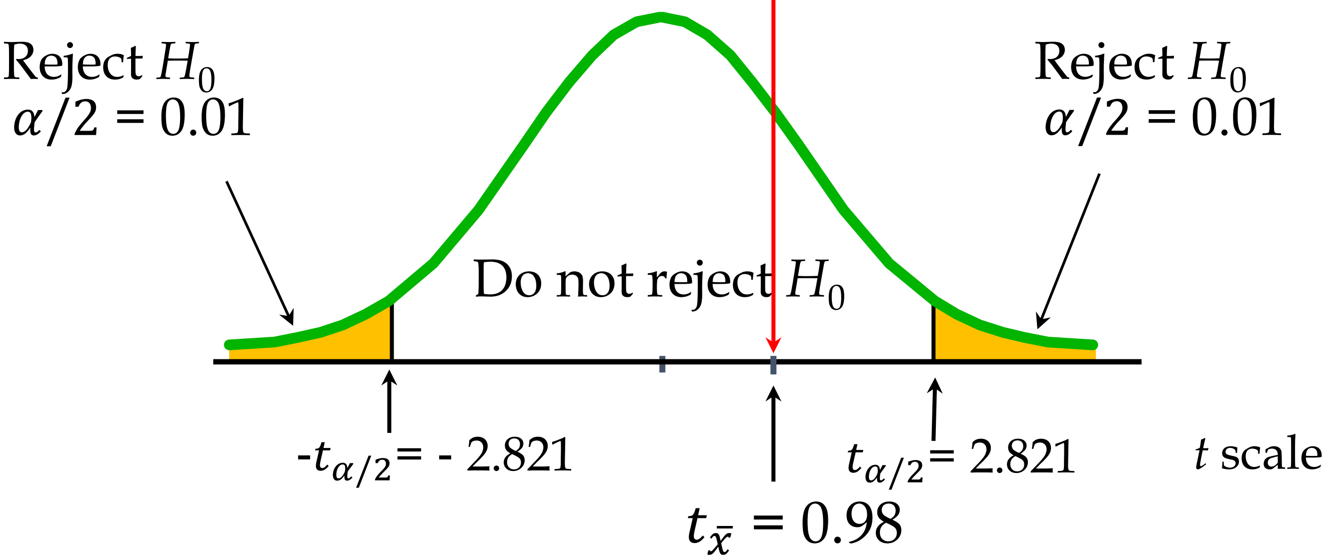
Two-Tail Hypothesis Test for the Population Mean (σ Is Unknown)

Step 5: Compare the *t*-test statistic and the critical *t*-score

- For a two-tail test, reject the null hypothesis if

$$|t_{\bar{x}}| > |t_{\alpha/2}|$$

- $t_{\bar{x}}$ =0.98 is not greater than $|t_{\alpha/2}|$ = 2.821⇒**do not reject H_0**



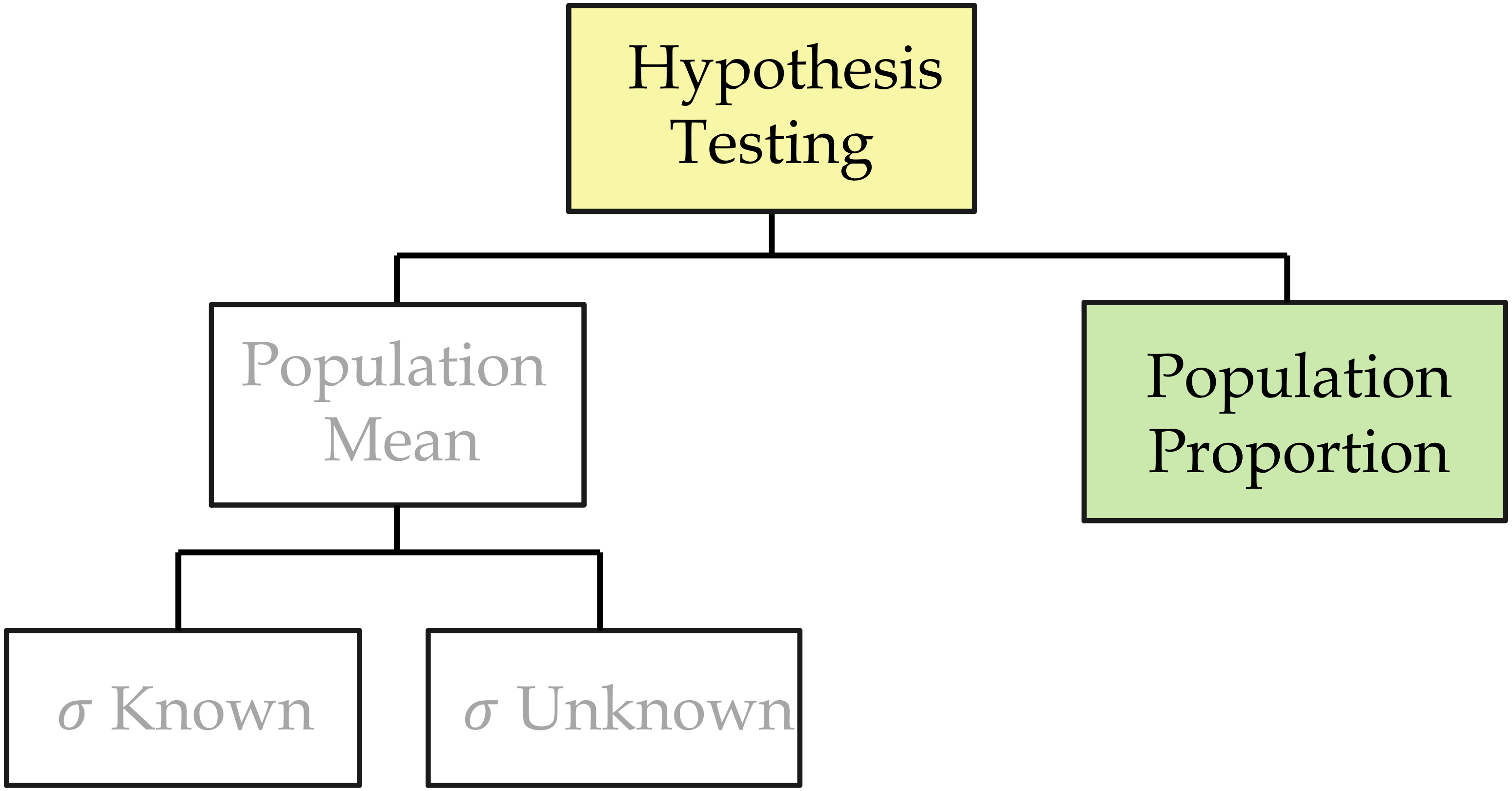
Two-Tail Hypothesis Test for the Population Mean (σ Is Unknown)

Conclusion

Our random sample of 10 viewers does not provide a sufficient evidence to reject the null hypothesis. Therefore, we have no evidence to conclude that the average hours in front of TV are different from 34.5 hours per week

Excel Exercise >>

Hypothesis Testing for the Population Proportion



Hypothesis Testing for the Population Proportion

E.g., we might test a hypothesis about the population proportion to answer the following questions:

- Whether more than 50% of physicians have been sued due to malpractice?
- Whether the proportion of those who receive coupons in the stores and actually use them later is greater than 10%?
- Whether the proportion of employers who gave holiday gifts to their employees is greater than 35%?
- Whether the proportion of those who live in their state of birth has increased?

Hypothesis Testing for the Proportion of a Population

Sample Proportion:

$$\bar{p} = \frac{x}{n}$$

Proportion data (sample) follow the binomial distribution, which can be approximated by the normal distribution if:

$$np \geq 5 \quad \text{and} \quad n(1 - p) \geq 5$$

where p = the population proportion
x = the number of observations of interest
n = the sample size

z-test Statistic for a Hypothesis Test for the Proportion

$$z_p = \frac{\bar{p} - p_{H_0}}{\sqrt{\frac{p_{H_0}(1 - p_{H_0})}{n}}}$$

where:

z_p = the z-test statistic for the proportion

$\bar{p}$ = the sample proportion

p_{H_0} = hypothesized value of a population proportion which is assumed to be true for the null hypothesis

n = the sample size

One-Tail Hypothesis Test for the Proportion

Example: The proportion of cell phone users with 4G contracts last year was $p = 0.62$. A Verizon executive thinks the proportion has increased this year.

- Suppose that from a random sample of 350 users, 238 have 4G contracts
- Test the hypothesis using $\alpha = 0.05$

The following slides show **steps** to complete this test

One-Tail Hypothesis Test for the Proportion

Step 1: Identify the null and alternative hypotheses

$$H_0: p \leq 0.62$$

$$H_1: p > 0.62$$

Step 2: Set a value for the significance level $\alpha \Rightarrow \alpha = 0.05$

Step 3: Calculate the appropriate test statistic

Find the sample proportion and the z-test statistic for the hypothesis test for the population proportion:

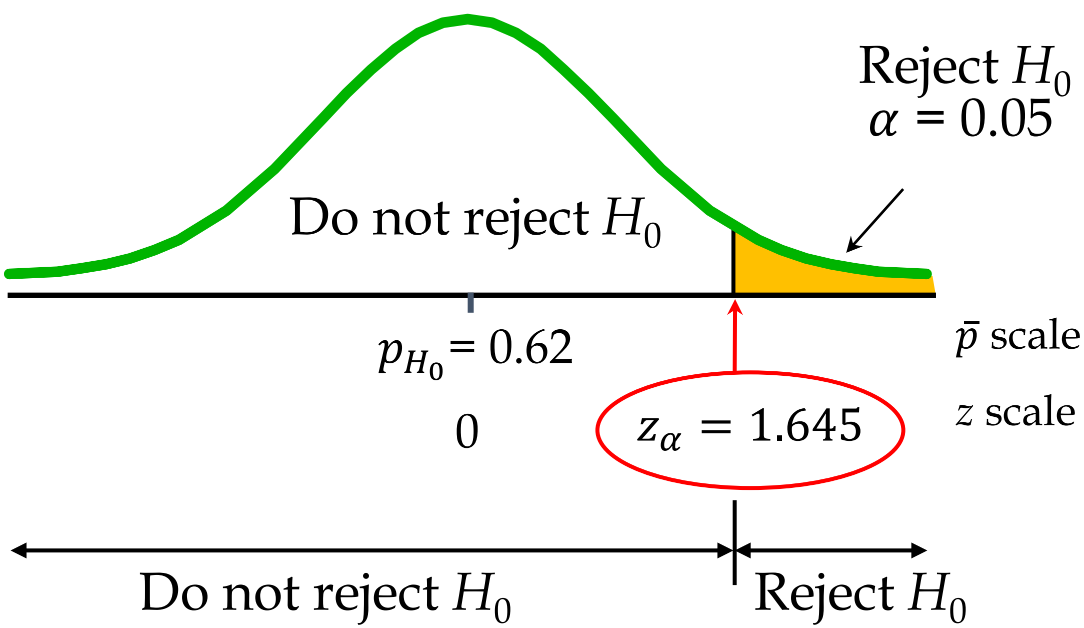
$$\bar{p} = \frac{x}{n} = \frac{238}{350} = 0.68$$

$$z_p = \frac{\bar{p} - p_{H_0}}{\sqrt{\frac{p_{H_0}(1-p_{H_0})}{n}}} = \frac{0.68-0.62}{\sqrt{\frac{0.62(1-0.62)}{350}}} = \frac{0.06}{0.026} = 2.31$$

One-Tail Hypothesis Test for the Proportion: Critical Value Approach

Step 4: Determine the appropriate **critical value**

- In the tests about the population proportion, we use the standard normal distribution to find the critical value and the *p*-value $\Longrightarrow$ find the critical *z*-score
- Since this is a one-tail test the entire area for $\alpha = 0.05$ is placed on the right side (upper tail) of the distribution:



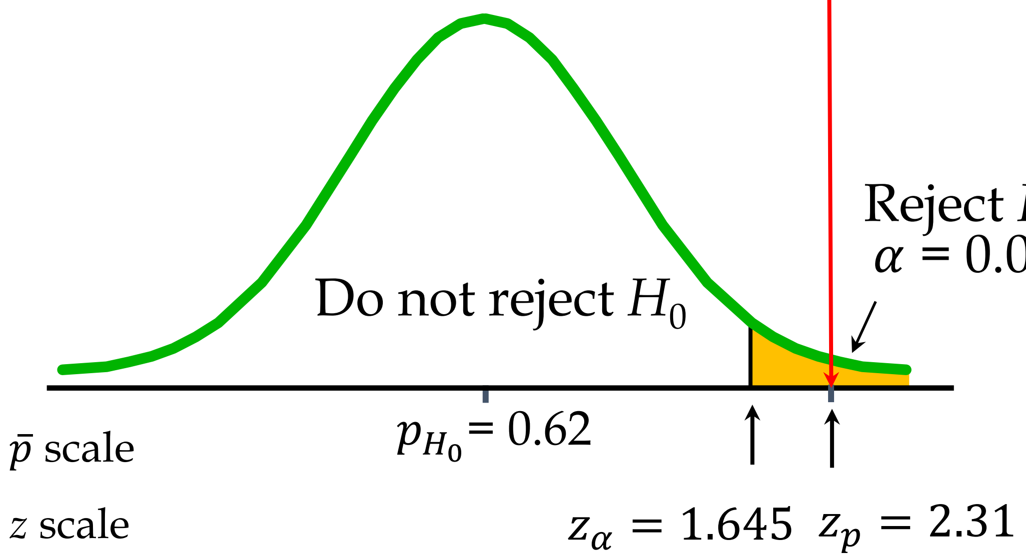
One-Tail Hypothesis Test for the Proportion: Critical Value Approach

Step 5: Compare the test statistic z_p and the critical value z_α

- For a one-tail (upper) test, reject the null hypothesis if

$$z_p > z_\alpha$$

- Here, z_p=2.31 is greater than z_α=1.645 ⇒ **reject H_0**

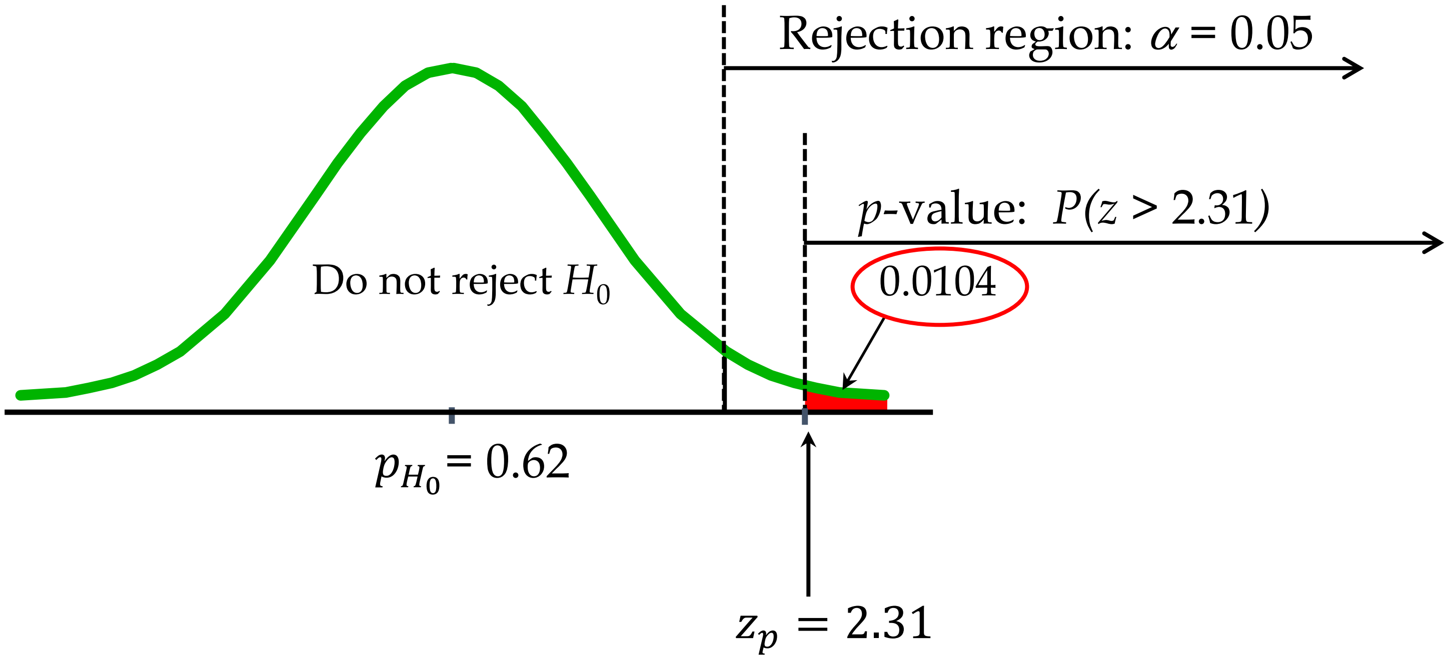


The *p*-value Approach to Hypothesis Testing for the Proportion

The *p*-value procedure for the hypothesis tests about the population proportion is identical to the procedure for the hypothesis tests for the population mean

p-value shows the probability of obtaining a sample result at least as unusual as the one observed, given that the null hypothesis is true

The p-value Approach to Hypothesis Testing for the Proportion



$$P(\bar{p} \geq 0.68) = P\left(z \geq z_p = 2.31\right) = 1 - 0.9896 = 0.0104$$

The p-value = 0.0104 is the probability of observing the proportion of users with 4G contracts in the sample equal to or greater than 68% if the actual proportion of users with 4G contracts is 62%

One-Tail Hypothesis Test for the Proportion

Conclusion

Since the p-value = 0.0104 < α = 0.05 $\Rightarrow$ **reject H_0**

Therefore, we can conclude at the 5% level of significance that the proportion of people with 4G contracts has increased above 62%

Excel Exercise >>

Exercise 9.56

The amount of text messages in the U.S. has been steadily declining since its peak in 2011 which was about 960 text messages per month per cell phone owner. This number declined steadily to reach 592 text messages in 2016. Suppose AT&T would like to test the hypothesis that the average number of text messages per month per owner has declined further since 2016. A recent random sample of 75 U.S. cell phone users was selected and the number of text messages sent last month was recorded. These data can be found in the Excel file **text_messages.xlsx** (*Excel Files* → *Ch 09*).

Using $\alpha = 0.1$, test if the average number of text messages sent per month per owner has decreased since 2016. For the hypothesis test:

- Use the critical value approach
- Use the *p*-value approach

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Excel Time: Exercise 9.50 (Modified)

According to Fidelity Investments, the average 401(k) account balance was \$99,900 in 2017. To test if this average has recently changed, suppose a sample of 30 401(k) plans was selected, and it was found that the average 401(k) balance was \$110,110 with the sample standard deviation equal to \$27,140.

Using $\alpha = 0.02$, does this sample provide enough evidence to conclude that the average 401(k) account balance has changed since 2017? For the hypothesis test:

- Use the critical value approach
- Use the p-value approach
- If $\alpha = 0.05$ has been used rather than $\alpha = 0.02$, would your conclusion differ?

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Excel Time: Estimating the p-value Using the Student's t-distribution

Excel's function T.DIST() can be used to find a p-value in a one-tail hypothesis test :

= **T.DIST**(x, degrees_of_freedom, cumulative)

- When we set x to the test statistic and cumulative = TRUE, the function provides the **area to the left** of the test statistic which is a desired p-value for the **lower tail test**.
- For the **upper tail test**, subtract the resulting value from 1.
- For the two-tail test, find the area to the left of the test statistic (if it's negative) or to the right of the test statistic (if it's positive), and multiply by 2.
- Recall that the degrees of freedom are found as $df = n - 1$.

Excel Time: Estimating the *p*-value Using the Student's *t*-distribution

Excel's function **T.DIST.2T** can be used to find the *p*-value for the two-tail hypothesis test :

= T.DIST.2T(*x*, degrees_of_freedom)

When we set *x* = **absolute value of the *t* test statistic**, the function provides the area in both tails of the *t*-distribution.

Excel Time: Estimating the *p*-value Using the Student's *t*-distribution

If you need an area to the right of the *t* test statistic,

- For the *p*-value in the upper tail test, or
- For the *p*-value in the two-tail test

then you can use **T.DIST.RT()** function:

= **T.DIST.RT**(*x*, degrees_of_freedom)

- When we set *x* to our test statistic, the function returns the **area to the right** of the test statistic which is a desired *p*-value for the **upper tail test**.
- If the value of the test statistic is positive in the two-tail test, then you can find an area to the right of the test statistic using =T.DIST.RT() and multiply it by 2.

Excel Time: *t*-distribution Functions

When using Excel to calculate probabilities from the *t*-distribution or find the *t*-value for a given probability, be careful to distinguish between function which have or do not have 2T in the end

2T stands for "two tails". Therefore,

- T.DIST.2T() ⇒ we input the absolute value of *t*, and the function returns the area in **both** tails, above *t* and below -*t*
- T.INV.2T() ⇒ we input the area in both tails of the *t* distribution, and the function returns the absolute value of *t* such that the area below –*t* and above *t* add up to the inputted probability

Summary of Excel Functions: Critical Value, II

Test	**Distribution**	**Value**	**Excel Function**
Two-tail	t	$t_{\alpha/2}$	= T.INV.2T(α, *df*)
			= ABS(T.INV($\alpha/2$, *df*))
			= T.INV(1- $\alpha/2$, *df*)
One-tail	t	t_{α}	= ABS(T.INV(α, *df*))
			= T.INV(1 - α, *df*)

Summary of Excel Functions: *p*-value, II

Test	**Distribution**	***p*-Value**
Two-tail	t	= T.DIST.2T(ABS(t), df) = 2 × T.DIST(t, df, 1) if $t < 0$ = 2 × (1-T.DIST(t, df, 1)) if $t > 0$ = 2 × T.DIST.RT(t, df) if $t > 0$
One-tail (*lower tail*)	t	= T.DIST(t, df, 1)
One-tail (*upper tail*)	t	= 1-T.DIST(t, df, 1) = T.DIST.RT(t, df)

Excel Time: Exercise 9.31 (Modified)

The number of long-term unemployed, defined as those out of work for more than 27 weeks, has been a detriment to the recent economic recovery. At its peak after the most recent economic recession, that rate reached 45.1% in July 2011 but has been steadily declining since to 40% in April 2018.

Government policy makers feel that this percentage has declined recently as the job market has improved. To test this theory, a random sample of 300 unemployed people was selected, and it was found that 96 were unemployed for longer than 27 weeks.

Using $\alpha = 0.01$, can the federal government conclude that the percentage of unemployed who have been out of work for more than 27 weeks has recently declined? For the hypothesis test:

- Use the critical value approach
- Use the *p*-value approach

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Excel Time: Exercise 9.16 (Extra Practice)

According to the 2016 Majoring in Money report by the government-lending institution Sallie Mae, 18- to 20-year-old college students have an average credit card balance of \$569. You want to find out if this number is lower now and collect data for a random sample of 40 college students. Their average credit card debt was found to be \$545 with a sample standard deviation of \$68.55.

Using $\alpha = 0.05$, does this sample provide enough evidence to conclude that the average credit card balance of 18- to 20-year-old students has decreased? For the hypothesis test:

- Use the critical value approach
- Use the p-value approach
- Does changing the value of α from 0.05 to 0.01 affect your conclusion? Why or why not?

Excel Time: Exercise 9.26 (Extra Practice)

An increased number of U.S. colleges have been using online resources such as Facebook and Google to research applicants. According to Kaplan Test Prep, 35% of admission officers indicated that they visited an applying student's social networking page in 2017. A random sample of 100 admissions officers was selected and it was found that 47 of them visit student's social networking pages.

Using $\alpha = 0.05$, does this sample provide support for the hypothesis that the proportion of admissions officers who visit an applying student's social networking page has increased since 2017? For the hypothesis test:

- Use the critical value approach
- Use the *p*-value approach

Excel Time: Exercise 9.64 (Extra Practice)

Small businesses are viewed as a backbone of the U.S. economy. Small-business owners are often described as entrepreneurs, requiring a variety of skills in order to be successful. A 2016 survey of small-business owners found that 47% had a college degree beyond a bachelor's degree. Suppose the Small Business Administration would like to test the hypothesis that the percentage of small-business owners with more than a bachelor's degree is different than 47%. The Excel file **SBA.xlsx** (*Excel Files → Ch 09*) shows the college degree of a recent sample of owners.

Using $\alpha = 0.05$, perform a hypothesis test to determine if the percentage of small-business owners with more than a bachelor's degree is different than 47%? For the hypothesis test:

- Use the critical value approach
- Use the *p*-value approach

Excel Time: Exercise 9.20 (Extra Practice)

Fannie Mae is a government-sponsored organization that was established in 1938 after the Depression to provide local banks with money from the federal government to be used for residential mortgages in an effort to increase homeownership rates. As lending standards have tightened during the most recent housing market crisis, credit scores for borrowers of approved mortgages have increased. In 2017, the average credit score for loans purchased by Fannie Mae was 745. A random sample of 35 mortgages recently purchased by Fannie Mae have shown the average credit score of 755 with a sample standard deviation of 21.

Using $\alpha = 0.035$, is there enough evidence to conclude that the average credit score for mortgages purchased by Fannie Mae has increased since 2017?